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Abstract

We consider a standby redundant model of two units under Difficult maintenance policy. We shall prove that under the condition that the mean life time of the system is much larger than the mean repair time and the maintenance time, the system reliability function will tend to be exponential one.

1. Introduction

In practice there are three maintenance policies defined as:

1. Economic maintenance; when the inspection moment is attained, the preventive maintenance performs only if there exist a reserve element in standby. If there is no element in standby, the preventive maintenance does not perform and the operative elements are operating usually up to the failure moment of one of them.

2. Moving maintenance; the preventive maintenance performs only if the reserve element is in standby. If the reserve element is under repair or preventive maintenance, then the preventive maintenance performs only after completion repair or preventive maintenance of the reserve element.

3. Difficult maintenance; in which the preventive maintenance performs independently on the state of the reserve element which may be in standby, repair or preventive maintenance.

In the case of standby redundant system with repair mentioned in [1] and [2], it is proved that the repairable distribution function will tend to exponential one when the mean repair time is very composed to the mean life time of the system. The general standby redundant system with maintenance, has been studied by several authers [3], [4], [5]. The limiting distribution function of the duplex system with preventive maintenance when the mean life time of the system is very large with respect to the mean duration of the repair and the mean time of inspection for policy I and II is studied by [6]. In this paper we study the limiting distribution function of the same system under policy III.

2. The limiting failure time distribution of the system

If the operation time T_0 of the main equipment is more greater than the time of repair T_R, and the time of inspection T_I i.e. $T_0 \gg T_R$, $T_0 \gg T_I$, then there exist $\nu, \varepsilon_{\nu_1}, \varepsilon_{\nu_2}$ such that

$$\bar{G}_\nu(x) = 1 - G_\nu(x) < \varepsilon_{\nu_1} \; ; \; \bar{V}_\nu(x) = 1 - V_\nu(x) < \varepsilon_{\nu_2} \; , \; x > 0$$

$$\text{or} \quad \lim_{\nu \to \infty} \bar{G}_\nu(x) = 0 \qquad (1)$$

$$\lim_{\nu \to \infty} \bar{V}_\nu(x) = 0 \qquad (2)$$

When $F(X)$, $G_\nu(X)$, $V_\nu(x)$ are the distribution functions for failure, repair and maintenance time of the element, the following theorem is true

Theorem 1. In the case of a constant time preventive maintenance policy i.e.

$$U(t) = \begin{cases} 1 & T \gg T \\ 0 & < T \end{cases} \qquad (3)$$

where T is of the same order as T_0, and

$$\alpha_\nu = F(T) \int_0^T \bar{G}_\nu(t)\, dF(t) + \bar{F}(T) \int_0^T \bar{V}_\nu(t)\, dF(t) \xrightarrow[\nu \to \infty]{} 0 \qquad (4)$$

Then for the "Difficult maintenance", the distribution function of the operating time of the system tends to exponential.

Proof:

The probabilistic integral equations for the system are

$$R(t) = \bar{F}(t)\bar{U}(t) + \int_0^t \bar{U}(x)\, R_1(t-x)\, dF(x)$$

$$+\int_0^t \overline{F}(x)\,R_2(t-x)\,dU(x)$$

$$R_1(t)=\overline{F}(t)\,\overline{U}(t)+\int_0^t R_1(t-x)\,G(x)\,\overline{U}(x)\,dF(x)$$

$$+\int_0^t R_2(t-x)\,G(x)\,\overline{F}(x)\,dU(x)$$

$$R_2(t)=\overline{F}(t)\,\overline{U}(t)+\int_0^t R_1(t-x)\,V(x)\,\overline{U}(x)\,dF(x)$$

$$+\int_0^t R_2(t-x)\,\overline{F}(x)\,V(x)\,dU(x)$$

Considering (3), the Laplace transform of the distribution function of the operating time of the system written as a function of $\alpha_\nu s$ is

$$R(\alpha_\nu s)=d_1(\alpha_\nu s)+d_2(\alpha_\nu s)+\frac{\lambda_1(\alpha_\nu s)\{d_1(\alpha_\nu s)+d_2(\alpha_\nu s)-1\}}{\lambda_2(\alpha_\nu s)}$$

where

$$\lambda_1(\alpha_\nu s)=[d_1(\alpha_\nu s)+d_2(\alpha_\nu s)+d_1(\alpha_\nu s)\{C_1(\alpha_\nu s)-C_2(\alpha_\nu s)\}$$
$$+d_2(\alpha_\nu s)\{b_2(\alpha_\nu s)-b_1(\alpha_\nu s)\}]$$

$$\lambda_2(\alpha_\nu s)=1-b_1(\alpha_\nu s)-C_2(\alpha_\nu s)+b_1(\alpha_\nu s)C_2(\alpha_\nu s)$$
$$-C_1(\alpha_\nu s)\,b_2(\alpha_\nu s)$$

$$d_1(\alpha_\nu s)=\int_0^T e^{-\alpha_\nu st}\,dF(t)$$

$$d_2(\alpha_\nu s)=e^{\alpha_\nu sT}\,\overline{F}(T)$$

$$b_1(\alpha_\nu s)=\int_0^T e^{-\alpha_\nu st}\,G_\nu(t)\,dF(t)$$

$$b_2(\alpha_\nu s)=\int_0^T e^{-\alpha_\nu st}\,V_\nu(t)\,dF(t)$$

$$C_1(\alpha_\nu s)=e^{-\alpha_\nu ST}\,G_\nu(T)\,\overline{F}(T)$$

$$C_2(\alpha_\nu s)=e^{-\alpha_\nu ST}\,V_\nu(T)\,\overline{F}(T)$$

Since

$$d_1(\alpha_\nu s)+d_2(\alpha_\nu s)\xrightarrow[\alpha_\nu\to 0]{}1 \qquad (5)$$

$$\frac{d_1(\alpha_\nu s)+d_2(\alpha_\nu s)-1}{\alpha_\nu}=-s\int_0^T \overline{F}\, e^{-\alpha_\nu st}\, dt \xrightarrow[\alpha_\nu\to 0]{} -sL \tag{6}$$

where $L=\int_0^T \overline{F}(t)\, dt$

$$\begin{aligned}\lambda_1(\alpha_\nu s)=&\int_0^T e^{-\alpha_\nu st}\, dF(t)+e^{-\alpha_\nu sT}\,\overline{F}(T)\\ &+e^{-\alpha_\nu sT}\,\overline{F}(t)\,\{G_\nu(T)-V_\nu(T)\}\int_0^T e^{-\alpha_\nu st}\, dF\\ &+e^{-\alpha_\nu sT}\,\overline{F}(T)\int_0^T e^{-\alpha_\nu st}\,[V_\nu(t)-G_\nu(t)]\,dF(t)\xrightarrow[\alpha_\nu\to 0]{} 1\end{aligned} \tag{7}$$

$$\begin{aligned}\lambda_2(\alpha_\nu s)=&[1-e^{-\alpha_\nu sT}\,V_\nu(T)\,\overline{F}(T)]\int_0^T e^{-\alpha_\nu st}\,\overline{G}_\nu(t)\,dF(t)\\ &+e^{-\alpha_\nu sT}\,G_\nu(T)\,\overline{F}(T)\int_0^T e^{-\alpha_\nu st}\,(1-V_\nu(t)\,dF(t)\\ &+e^{-\alpha_\nu sT}\,F\,(1-V_\nu(T))+\alpha_\nu s\int_0^T \overline{F}(t)\,e^{-\alpha_\nu st}\\ &+e^{-\alpha_\nu sT}\,\overline{F}(T)\,\{V_\nu(T)-G_\nu(T)\}\int_0^T e^{-\alpha_\nu st}\,dF(t)\end{aligned}$$

Since $V_\nu\ (T) = G_\nu\ (T) = 1$; then we have

$$\frac{\lambda_2(\alpha_\nu s)}{\alpha_\nu}\xrightarrow[\alpha_\nu\to 0]{} 1+sL+O(\alpha) \tag{8}$$

It is evident from (5), (6), (7) and (8) that

$$R(\alpha_\nu s)\xrightarrow[\alpha_\nu\to 0]{}\frac{1}{1+sL}$$

which completes the proof of the theorem.

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